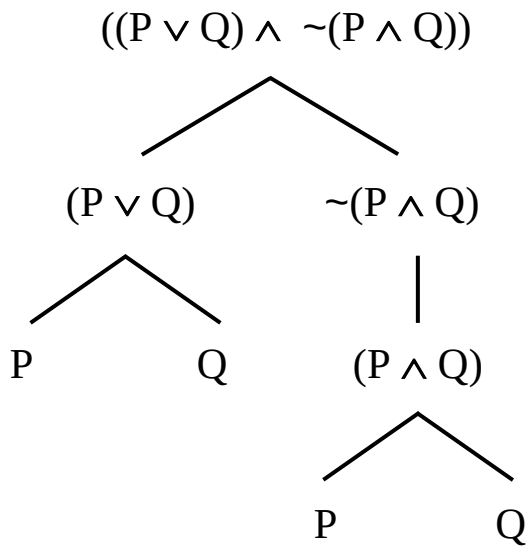
2.16.1. Semantics and Construction Problems

A. Based on the construction tree for the following sentence, state the **four mistakes** made in the truth table steps for that sentence, given below.



P	Q	R	(P ∧ Q)	~P	~Q	~(P ∧ Q)	((P ∨ Q) ∧ ~(P ∧ Q))

B. Beginning with the truth values for "P" and "Q" from the following valuation, use the semantic rules to figure out the **truth value of each larger sentence** appearing in the construction tree. (*For example: since "Q" is* 0 *in this valuation, the Negation Rule says "~Q" is* 1.)[1]

Negation Rule

▲	~▲
1	0
0	1

Conjunction Rule

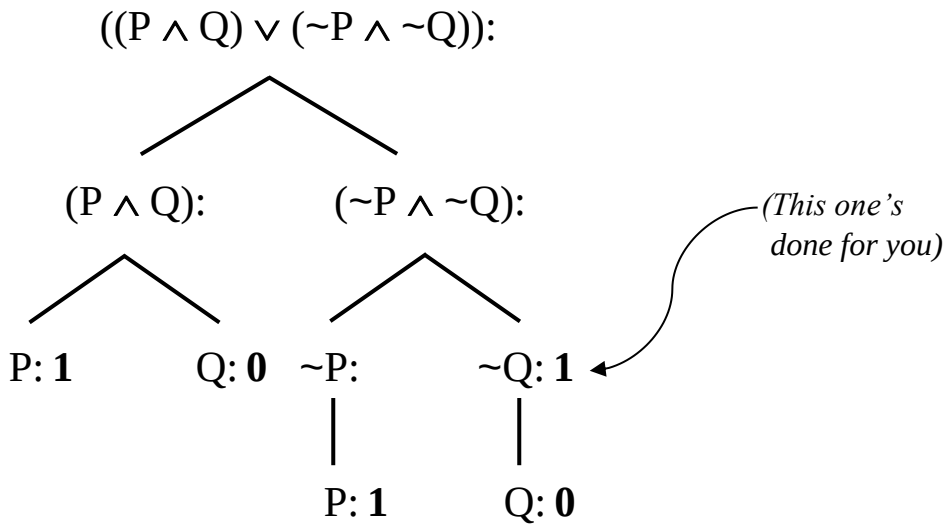
●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

Disjunction Rule

●	▲	(● ∨ ▲)
1	1	1
1	0	1
0	1	1
0	0	0

Valuation:

P	**Q**
1	0



The same thing would be written in **truth table** format like this:

P	Q	(P ∧ Q)	~P	~Q	(~P ∧ ~Q)	((P ∧ Q) ∨ (~P ∧ ~Q))
1	0			1		

[1] Seeing how the same valuation can depicted in either a (vertical) tree or a (horizontal) truth table, it's clear why we prefer truth table format: writing four valuations **in tree form** would take up **two pages**, while writing them **in table form** takes only **five lines** (including the sentences across the top).

C. For each of the sentences listed below, state **that sentence's truth value** in the **valuation** given here.

Negation Rule

▲	~▲
1	0
0	1

Conjunction Rule

●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

Disjunction Rule

●	▲	(● ∨ ▲)
1	1	1
1	0	1
0	1	1
0	0	0

Valuation:

P	**Q**	**R**	**S**
1	1	0	0

1. (Q ∨ R)
2. (P ∧ Q)
3. (P ∧ R)
4. ~(P ∧ R)
5. (P ∨ R)
6. (P ∧ (Q ∨ R))
7. ~S
8. (~S ∧ R)
9. ((~S ∧ R) ∨ (P ∨ R))
10. ((~S ∧ R) ∧ (P ∨ R))

D. **Advanced Semantic Fun**. Using the same valuation as in (C), state for each of the following sentences **whether its truth value can be determined** in that valuation. If it can, **state the sentence's truth value** in that valuation. (*The truth values for "*T*" and "*U*" are not given.*)

1. (S ∧ T)
2. (Q ∧ T)
3. (~Q ∧ T)
4. ~(R ∧ U)
5. (~Q ∨ (R ∧ T))
6. (~Q ∨ (U ∧ T))
7. ((~Q ∧ T) ∨ (R ∧ U))
8. ~((~Q ∧ T) ∨ (R ∧ U))

E. **Translate** each of the following English sentences into the formal language (using the **same translation key** for all three sentences). Then **build truth tables** for those three formal sentences, to decide:

(i) Is one sentence here the **negation** of another sentence here?
(ii) Are any two of these sentences **logically equivalent**?

1. Gambling on logic is either legal or satisfying.
2. Gambling on logic is either illegal or unsatisfying.
3. Gambling on logic is neither legal nor satisfying.